Single item stochastic lot sizing problem considering capital flow and business overdraft

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Abstract

This paper introduces capital flow to the single item stochastic lot sizing problem. A retailer can leverage business overdraft to deal with unexpected capital shortage, but needs to pay interest if its available balance goes below zero. A stochastic dynamic programming model maximizing expected final capital increment is formulated to solve the problem to optimality. We then investigate the performance of four controlling policies: (R, Q), (R, S), (s, S) and $(s,\ \overline{Q},\ S)$; for these policies, we adopt simulation-genetic algorithm to obtain approximate values of the controlling parameters. Finally, a simulation-optimization heuristic is also employed to solve this problem. Computational comparisons among these approaches show that policy (s, S) and policy $(s, \overline{Q}, S)$ provide performance close to that of optimal solutions obtained by stochastic dynamic programming, while simulation-optimization heuristic offers advantages in terms of computational efficiency. Our numerical tests also show that capital availability as well as business overdraft interest rate can substantially affect the retailer's optimal lot sizing decisions.

1 Introduction

Overdraft is widely used by many companies to prevent capital shortage. It is necessary and important for a manager to take capital flow and external financing into account when making operational decisions. Our contributions to the lot sizing problem are the following:

- We incorporate capital flow and one kind of external financing, i.e. overdraft, in the stochastic lot sizing problem and formulate a stochastic dynamic programming model to obtain optimal solutions.

- We discuss four inventory controlling policies for this problem and use simulation-genetic algorithm to obtain approximate values of the controlling parameters.

- We introduce a simulation-optimization heuristic inspired by the approach.

- We conduct a comprehensive numerical study to compare stochastic dynamic programming, simulation-genetic algorithm and simulation-optimization heuristic.

2 Problem description

All the notations adopted in this paper is listed in Table 1. In our problem, demand is stochastic and non-stationary. For each period t, its demand is represented by D_t, which is a non-negative random variable with probability density function f_t, cumulative distribution function F_t, mean μ_t, variance σ^2. Random demand is assumed to be independent over the periods. Unmet demand in any given period is back ordered and satisfied as soon as the replenishment arrives. Excess stock is transferred to next period as inventory and the sell back of excess stock is not allowed.

We assume the initial capital quantity of the retailer is B_0; order delivery lead time is zero; selling price of the product is p and the retailer receives payments only when the products are delivered to customers; a fixed cost a is charged when placing orders, regardless of the ordering amount, and R_t is a 0-1 variable to determine whether the retailer makes order at period t; a variable cost v is charged on every ordering unit; end-of-period inventory level for period t is I_t, and we set I_t^+ to represent $\max\{I_t, 0\}$ and I_t^- to represent $\max\{-I_t, 0\}$; a variable inventory holding cost h is charged on every product unit carried from one period to the next; per unit stock-out penalty cost is π; at the beginning of each period t, its present capital is B_{t-1}, if its initial capital is below zero, the retailer has to pay interests with a rate of b.

End-of-period capital B_t for period t is defined as its initial capital B_{t-1}, plus payments by customers for satisfied demand of this period, minus the payments to suppliers for orders made in this period and this period's fixed ordering cost, holding and backorder costs, and minus the interest paid if its initial capital is negative. It can be represented by the following equation.

$$B_t = B_{t-1} + p\min\left\{D_t + I_{t-1}^-, Q_t + I_{t-1}^+\right\} - \left(vQ_t + aR_t + hI_t^+ + \pi I_t^-\right) - b\max\{-B_{t-1}, 0\} \tag{1}$$

The actual sales amount in period t is $\min\left\{D_t + I^-_{t-1}, Q_t + I^+_{t-1}\right\}$, where $D_t + I^-_{t-1}$ is demand plus backorder in period t and $Q_t + I^+_{t-1}$ is the total available stock in period t.

For the final capital of the retailer in the whole planning horizon, we defined it as the end-of period capital B_T, minus the interest paid if B_T is negative, which is:

$$B_{T+1} = B_T - b\max\{-B_T, 0\} \tag{2}$$

We use a tilde above the parameter to represent its expected value. Our aim is to find a replenishment plan that maximizes the expected final capital increment, i.e. $\tilde{B}_{T+1} - B_0$.

3 Results and discussion

6 periods with different demand patterns are adopted for experiments and there are 640 numerical cases in total, our computation results show that policies (s, S) and $(s, \overline{Q}, S)$ solved by genetic algorithm, in general perform better than other approaches (RMSE: 3.17 and 3.25, respectively; MAPE: 5.68% and 5.59%, respectively), followed by policy (R, S) (RMSE: 6.28, MAPE: 26.72%), simulation-optimization heuristic (RMSE: 13.60, MAPE: 53.67%) and policy (R, Q) (RMSE: 15.90, MAPE: 66.63%). Considering the confidence levels, performance of policy (s, S) and policy $(s, \overline{Q}, S)$ are essentially identical. For the four controlling policies, it can be concluded that their performance is related with their flexibility. Since policy (s, S) and policy $(s, \overline{Q}, S)$ are based on "dynamic uncertainty" strategy, which is most flexible, they perform best for the problem, while the least flexible policy (R, Q) has worst performance. It is however surprising that enforcing a maximum order quantity $\overline{Q}$ does not seem to be beneficial, and that an (s, S) policy with parameters carefully selected seems to provide competitive performances.

The performance of different approaches does not seem to be affected by different parameter levels under the criterion RMSE; however, it is affected by the margin of product — selling price and unit variable ordering cost under the MAPE criterion. Finally, the performance of the simulation-optimization heuristic varies substantially across different demand patterns.

In terms of computation times, the simulation-optimization heuristic runs faster than genetic algorithm, with average computation time less than one second (0.04s). Among the policies solved via genetic algorithm, policy (R, S) runs fastest (42.31s), followed by policy (R, Q) (46.91s), policy (s, S) (184.88s), policy $(s, \overline{Q}, S)$ (194.39s).

Notations	Description
Indices	
t	Period index, $t = 1, 2, \ldots, N$
Problem parameters	
B_0	Initial capital
I_0	Initial inventory level
I_t^+	$\max\{I_t, 0\}$
I_t^-	$\max\{-I_t, 0\}$
p	Product selling price
a	Fixed ordering cost
v	Unit variable ordering cost
h	Unit inventory cost
π	Unit penalty cost for back orders
b	Interest rate for minus capital
M	A big number
Random variables	
D_t	Random demand at period t with probability density function $f_t(D_t)$, cumulative distribution function $F_t(D_t)$, mean μ_t, variance σ^2
State variables	
I_t	End-of period inventory for period t, we assume $I_0 = 0$
B_t	End-of period capital for period t
Decision variables	
Q_t	Ordering quantity at the beginning of period t
R_t	whether the retailer orders at period t
S_t	Order up to level at the beginning of period t, and $S_t = I_{t-1} + Q_t$
s_t	Threshold of the inventory level for (s, S) policy

Table 1: Notations adopted in our paper